Misallocation Implications of Security and Collateral Value of Land

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Abstract

We explore two models of land misallocation resulting from insurance and credit market imperfections respectively. We examine if they can explain misallocation resulting from negative wealth effects in marginal willingness to pay for land by farming households, as observed in eastern India. We show this pattern is consistent with a model with missing markets for insurance against weather shocks, where farmers value the security provided by access to cultivable land against uncertain costs of food subsistence requirements. The security value of land is shown to be decreasing in ability and in wealth among agents of below average ability, under weak assumptions on risk preferences and patterns of heterogeneity in the population. Equilibria in the land market always feature misallocation owing to resulting biases in favor of low ability poor farmers. In contrast, in the model where land is valued as a source of collateral we find that the collateral value of land is increasing in ability and wealth. If land is perfectly collateralizable, there is no land misallocation in equilibrium. The contrasting predictions of the two models concerning the pattern of wealth effects are potentially testable, and suggest that the security value of land poses a more potent channel of persistence of land misallocation.

1 Introduction

The macro-development literature measures misallocation by departure of factor allocations from a first-best benchmark of productive efficiency. This literature throws light on variations

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in aggregate productivity across regions, countries or periods attributable to various kinds of frictions facing private agents. This gives rise to questions about the *sources* of such frictions, which has important implications on the *welfare* impact of policies affecting them as opposed to the focus on productive efficiency.

A particular focus of interest in this literature has been to explore the reason for wide variations in agricultural productivity across developing and developed countries (Lagakos, Gollin and Waugh 2014). Some of the literature provides evidence of the role of government restrictions on operation of land markets in preventing allocation of land to more productive farmers in China, Ethiopia and Malawi (Adamopoulos et al 2022, Chen, Restuccia and Santaeulalia-Lopis 2022, 2023). Foster and Rosenzweig (2022) emphasize the role of labor market transaction costs and scale economies in explaining low productivity in middle size farms compared to small and large farms in India.

Absence of private property rights or land titles, existence of land regulations and frictions on the land market may account for the persistence of land misallocation, by reducing the extent to which high ability farmers are able to purchase or rent land from low ability farmers. Yet land market regulations or frictions cannot constitute the entire story for countries with private property rights. In various parts of India for instance detailed studies provide evidence of active land sales. Bardhan et al 2014 show 26% of households in rural West Bengal engaged in land sales and 23% in purchases between 1968-2004; land sales accounted for a drop in land owned per household by 0.55 acres (out of a net fall of 1.37 acres during this period, the rest being accounted by household division, land reforms and gifts).[1] Moreover, they show that the incidence of land transactions were consistent with a model of how incentives to buy and sell land are affected by land reforms and changes in household demographics. Besley et al 2016 show similar evidence regarding the effect of tenancy reforms on land distributions operating via the land market in four South Indian states.

The question then is what prevents further land transactions that eliminate misallocation? One possible explanation is the existence of credit market imperfections. Standard models of credit market imperfections where credit access depends on financial wealth (e.g., Buera, Kaboski and Shin 2011, Buera and Shin 2013 or Moll 2014) predict that valuations placed by households (i.e., marginal willingness to pay) for land are increasing in wealth and ability. Misallocation could result owing to wealth effects which prevent high ability poor farmers from acquiring lands owned by wealthy farmers of lower ability. Alternatively, if for some reason land values are decreasing in wealth, small farmers who are poor may want to hold on to their land despite being less productive than large farmers. There is

[1] See Table A2 in the Online Appendix to Bardhan et al 2014.

some evidence of negative wealth effects in land valuations in rural West Bengal: Ghatak et al 2013 found that households with lower wealth were significantly more likely to refuse above-market prices offered by the government to acquire their land, controlling for quantity and quality of land owned, and a host of other household characteristics.[2]

This paper explores two potential explanations for land misallocation resulting from such negative wealth effects. Land could be valued for reasons other than the income it generates: e.g., it may be a source of security or collateral. Poorer households could be more concerned about uncertain shocks which adversely affect incomes or raise consumption needs so they could value land more if it helps insure against such shocks. Alternatively land which constitutes collateral that expands access to credit and therefore opportunities to invest in non-agricultural occupations or children's education would likely be more valued by poor credit-constrained households. There is considerable descriptive evidence of both sources of land value. For instance interviews conducted by Ghatak et al 2013 of households refusing compensation revealed one of the important reasons was their concern for increased exposure to price risk of subsistence crops that would result if they gave up their land.[3] The role of land as a form of collateral has been emphasized in the well-known work of Hernando de Soto (2000) as one of the benefits of land titling programs for the poor. While there is a substantial empirical literature on effects of land titling programs on land productivity and credit access (e,g,, see references cited in Manisheva 2022), there appears to be a scarcity of analyses of the implications of the collateral role of land on how land valuations vary with wealth and misallocation patterns that consequently result.

The first model we study explores the security value of land. We construct a heterogeneous agent incomplete market GE model with two dates, and two physical commodities: a food (subsistence) crop F, and a non-food manufactured good M. Production of food requires land and effort of the owner-cultivator, exhibits diminishing returns to land while required labor effort is proportional to area cultivated. Time not spent in cultivation is allocated to producing the M good, whose production requires only labor and subject to constant returns.[4] Uncertain rainfall shocks affect crop yields, and thereby the price of food.

[2]The study was conducted in Singur, West Bengal, one of the sites where 40% of farming households refused to accept above-market compensation for lands acquired by the government for a car factory project. Wealth was proxied by nonagricultural, non-labor income and whether land was inherited. Controls included plot characteristics (irrigation, elevation, location, cropping intensity), education of household head, total land owned, whether the owner had selling rights, and occupational composition of household members.

[3]Other cited reasons included unemployment risk, increased requests from relatives and friends for loans and gifts, and their own temptation to over-spend from the cash payments. The working paper version of Ghatak et al 2013 (https://personal.lse.ac.uk/ghatak/singur.pdf) and Ghatak and Banerji 2009 report some of these discussions.

[4]This is a simplifying assumption as our main focus is on the allocation of land across farming agents, rather than between agriculture and industry. An interesting extension of the model would be to incorporate

Agents differ in farm ability, endowments of land and financial wealth (storable M good stocks). As in standard models of incomplete markets, land assets are traded at the initial date, followed by productive activities. At the second date, rainfall shocks are realized, and commodity spot markets open.

Land ownership is valued both for its productivity and insurance roles, with the security value of land of each agent depending on their risk attitudes in the absence of Arrow-Debreu securities against rainfall shocks. Land provides security for the following reason. The spot market demand for food at the second date is price-inelastic, since food is a necessity, accounting for the bulk of subsistence needs. Hence food prices and household food expenditures rise when there is a drought, limiting resources available for consumption of the industrial good. Land cultivation rights provide partial insurance, as crop revenues rise during a drought. Land therefore tends to be more highly valued by risk-averse agents.

The key question we examine is how the security value of land varies with agent ability and wealth.[5] We characterize land misallocation arising in any market clearing equilibrium where ex ante price expectations are fulfilled. While agents are heterogeneous in their ability and endowments (with minimal restrictions on the distribution of these endowments), we discipline the model by assuming all agents share identical risk attitudes, represented by a common von Neumann Morgenstern utility function.

The nature of misallocation turns out to depend on how risk aversion varies with wealth. With non-decreasing absolute risk aversion (NDARA) , the distortions generated by land security biases the allocation in favor of high ability, high wealth agents, just as in the standard model of imperfect credit markets (Proposition 2). But if risk attitudes are represented by constant relative risk aversion (CRRA), distortions are in the opposite direction, always biased in favor of lower ability agents, and among a set of low-ability agents always biased in favor of poorer agents (Proposition 3). Within a set of high ability agents, they are biased in favor of wealthier agents, but this bias is dominated by that favoring the poorest, low ability agents. Remarkably, these results obtain irrespective of the level of relative risk aversion, and of the distribution of initial endowments.[6]

a more realistic model of the manufacturing sector, which is subject to scale economies and may require some land as an input aside from capital and labor. Such a model would be suitable to study questions relating to structural transformation and how it is affected by the process of land acquisition from farmers by manufacturing firms.

[5]One complication is that the latter is partially endogenous, since the land price is needed to value land endowments vis-a-via financial wealth. However, agents owning less land and financial wealth can unambiguously be identified as 'poorer', providing a partial ranking of wealth.

[6]The only restriction we impose on the latter is that all agents own a certain minimum level of food stocks, that ensures they are not liquidity constrained while purchasing land at the first date. This assumption is needed to abstract from traditional credit market imperfections, in order to focus on the security value of land *per se*.

Compared to a first-best setting the equilibrium results in a farm size distribution biased in favour of low ability, low wealth agents operating small low productivity farms. Therefore the security-based model generates a pattern of misallocation consistent with the standard finding of lower productivity and wealth of small landowners compared with large landowners, as well as negative wealth effects in marginal land valuations observed in the West Bengal study. Moreover, the equilibrium results in food prices that are higher on average and exhibit greater volatility. The equilibrium is constrained Pareto-suboptimal, applying more general results of Geanakoplos and Polemarchakis 1986 for incomplete market economies which apply to our setting.[7] The underlying intuition for this result is that in the decentralized market equilibrium, low ability agents ignore pecuniary external benefits for the entire economy (lower food prices and price volatility) resulting from sale of their land to high ability agents.

Section 3 turns to a model of the collateral value of land. It differs from standard models of credit market imperfections in one respect: land rather than financial wealth serves as collateral. Moreover, credit is valued by wealth constrained agents to expand the scale of working capital in a parallel trading activity (rather than in farming itself which is assumed to not require any working capital). The returns to the trading activity are decreasing in scale and do not depend on ability. To highlight the contrast with standard models, we assume financial wealth cannot be used as a source of collateral, perhaps because it can be diverted by defaulting borrowers. Instead land serves as collateral, because defaulting borrowers cannot 'run away with the land'. This allows a borrower to pledge a certain fraction of the return to land (i.e, standing harvest of the crop) to lenders in the event of default. We examine the resulting implications for how agents value land, and whether it differs from the previous version where land is a source of security. In this version there is no production risk, and there is a single physical good. Otherwise the framework is the same: agents have heterogeneous farming abilities, endowments of land and financial wealth. Land is traded at an initial date, followed by cultivation and trade activities, with resulting (deterministic) outputs consumed at a subsequent date.

In this model, agents choose land size to maximize the sum of the net returns to cultivation and to trading activity. This requires them to evaluate the marginal cultivation returns and marginal collateral value of land, where the latter equals the product of the rate of return to trading activity and the marginal expansion in trade activity made possible by an increase in land owned. In contrast to the security value of land, the marginal collateral value of land turns out to be non-decreasing in both wealth and farming ability of the

[7] See also Stiglitz 1982 for analogous arguments for constrained inefficiency of stock markets when insurance markets are incomplete.

borrower (as in standard models of financial collateral). It is increasing in farming ability because the extent to which land collateral relaxes credit constraints is increasing in ability (which raises the total crop output expected, and therefore the amount pledged to lenders). If 100% of crop output can be pledged, the marginal collateral value of land has the same sign as its marginal cultivation returns, so there is no misallocation in equilibrium. In this case the collateral role of land effectively overcomes the effect of imperfect credit market completely. In the more realistic case where less than 100% can be pledged, the marginal collateral value of land is smaller than its marginal cultivation return. In equilibrium the land size chosen by wealth constrained agents trade off a negative marginal collateral value of land against a positive marginal cultivation return. The marginal collateral role of land is larger for wealthier agents, since such agents operate closer to the first-best scale of trading activity. So they are less motivated to expand land size in order to borrow more.[8]

The main takeaway message of this paper is that models with insurance imperfections have characteristics quite distinct from models of credit market imperfections. The results suggest that the security value of land poses a more potent source of persistence of land misallocation. Which model is more applicable in a given setting is an empirical question. Our analysis shows that the two models generate contrasting predictions, thereby providing a way to test their relative validity empirically. They also have distinct implications for effectiveness of different policies on misallocation and welfare. If the security model is more appropriate, it would suggest the role of policies providing insurance against local weather shocks which may generate welfare gains both directly as well as indirectly by inducing less misallocation which would raise agricultural productivity and lower food price volatility. If on the other hand credit imperfections seem to be the underlying mechanism, policy makers may want to focus instead on ways to enhance credit access for poor but high ability farmers.

While related literature on credit market imperfections is vast, there is relatively little literature on insurance market imperfections. Karlan et al 2014 test for the significance of insurance imperfections relative to credit imperfections in the context of fertilizer application by African farmers. In the context of Indian agriculture Cole et al 2013 and Mobarak and Rosenzweig 2013 provide evidence of barriers to household risk management and explore implications of provision of rainfall index insurance. On implications of insurance imperfections for land policy, Baland and Francois 2005 provide an interesting model of adverse implications of privatization of common lands on consumption insurance of poor households which offset welfare gains from increased productive efficiency. They describe a number of case studies in India consistent with these predictions. Related empirical evidence on the links between landownership and insecurity include the following. Using data from a 2006

[8]Specifically, the marginal collateral value is increasing in wealth because it is less negative.

household land survey in Vietnam on land sales and transfers legalized by a 1993 land reform, Promsopha 2015 finds that among households that transferred land during 2001-06, those with more stable incomes were more likely to sell rather than transfer through some other means (rentals, loans or gifts). Rammohan and Pritchard 2014 provide evidence from Myanmar that households owning agricultural land were less prone to food insecurity, after controlling for incomes from various sources and various household characteristics.[9]

2 Security Value of Land

2.1 Assumptions

Consider an economy with either a continuum of agents, or a finite number of agents that act as price-takers: either formulation will be included (modulo some technical details that we shall gloss over, involving different expressions for average or aggregate demand functions). We shall place almost no restrictions on the joint distribution over heterogenous types (abilities and endowments) which can either be atomless or exhibit a finite support. Irrespective of these details, it will suffice for us to study how land demand patterns vary across agents of different types at any given set of prices.

There are two consumption goods: F (food) and M (manufactured). There are two production sectors: agriculture and industry which respectively produce F and M. Agents divide their time between these two sectors at date 0, and production occurs between two dates 0 and 1. Food is produced using land and (farm) labor in fixed proportions, with remaining time spent producing the M good which uses only labor. Land size is chosen by each agent and traded on a frictionless land market. Every agent supplies one unit of labor inelastically.

Production in the farm sector is subject to diminishing returns, while the M-sector has constant returns to scale. Agents have heterogenous farming abilities and wealth. They are equally productive in manufacturing. Weather shocks affect production of food, but not the manufactured good. The M-good is the numeraire; the constant productivity of workers in this sector is normalized to unity. Food production depends both on the ability of the agent (denoted by a) and the weather (denoted by w which takes two possible values: d (drought) and n (normal)).

Land size varies continuously over the unit interval for each agent: $l \in [0, 1]$, and an ability

[9]Less directly relevant is Knippenberg et al 2020 who show Ethiopian households with more fragmented landholdings (caused by inheritance and egalitarian state reallocations) experienced less food insecurity, owing to the greater opportunity to diversify crop risk. Land fragmentation thus provides another dimension where food security advantages offset losses in productive efficiency.

a agent farming l units of land produces $aA_w l^{1-\alpha}$ units of food in state w, where $\alpha \in (0,1)$ reflecting diminishing returns to land size, and A_w is a state dependent productivity shock satisfying $A_n > A_d$. Time devoted to farming is proportional to farm size, allowing the agent to allocate $1-l$ units of time to the M-sector and produce $1-l$ units of the M-good.

At date 0 the weather realization is uncertain; all agents share the same beliefs over the likelihood of drought. In a first-best complete market version of this economy, there would be Arrow-Debreu securities tradeable at date 0, allowing agents to purchase insurance in the form of date 1 contingent commodities. In the second-best economy that we study, such markets are absent. Instead, agents can trade in land at date 0. Food is perishable and cannot be stored, while the M good can be stored costlessly.

Agents do not consume at date 0; all consumption takes place at $t = 1$ after the state has been realized, outputs have been produced and traded on spot commodity markets. On the spot market agents share a common Stone-Geary utility function $\frac{1}{\zeta}(c_F - s)^\gamma (c_M)^{1-\gamma}$, where c_k denotes consumption of good $k = F, M$ and s is a minimum subsistence food requirement, $\gamma \in (0,1)$ and $\zeta \equiv \gamma^\gamma (1-\gamma)^{1-\gamma}$. Let p_w denote the price of food in state w. An agent trades on the date 1 spot market, with endowments that equal their respective outputs produced between dates $0, 1$, plus stocks of the M-good remaining after trading on the date 0 asset market. Hence the realized income Y_w of an agent with ability a, land size l and stock m_0 of the M good in state w at date 1 equals

$$Y_w = ap_w A_w l^{1-\alpha} + (1-l) + m = ap_w A_w l^{1-\alpha} - (1+P)l + 1 + W_0 \tag{1}$$

where $W_0 = m_0 + Pl_0$ denotes the value of the agents initial endowments of the M-good and land valued at the market price P. Given price-taking behavior, each agent's initial wealth is given, so an agent is characterized by two attributes: ability a and wealth W_0.

Risk attitudes at date 0 are represented by a common Neumann Morgenstern utility function V defined over spot market indirect utility. which is strictly increasing and strictly concave. At date-1 state w, an agent with income $Y_w > p_w.s$ will consume $s + \gamma \frac{(Y_w - p_w.s)}{p_w}$ units of food, and $(1-\gamma)(Y_w - p_w.s)$ units of the M-good, ending up with indirect utility

$$u_w \equiv \frac{Y_w - p_w.s}{p_w^\gamma}. \tag{2}$$

The corresponding date-0 expected utility will then be

$$W = \sum_{w=d,n} f_w V(u_w) \tag{3}$$

where f_w denotes the probability of state w.

Everyone takes prices as given on asset and spot markets. We study competitive equilibria in this economy, where (as in Radner 1972) all agents have perfect foresight at date 0 regarding the realization of date 1 commodity prices.

Agent heterogeneity is represented by a combination of farm ability a, initial endowments of land l_0 and M-good stocks m_0, i.e, by a three dimensional type vector (a, l_0, m_0). When agents trade in land at date 0 taking P the price of land as given, we can replace this by the two dimensional ability-wealth type vector (a, W_0) where $W_0 = m_0 + Pl_0$. We restrict the support of the type distribution to a compact set, where ability lies in an interval $\mathcal{A} \equiv [\underline{a}, \bar{a}]$, land endowment in $\mathcal{L} \equiv [\underline{l}_0, \bar{l}_0]$ and the M-good endowment in $\mathcal{M} \equiv [\underline{m}_0, \bar{m}_0]$. At land price P, this implies that the ability-wealth type of agents lies in the compact set $\mathcal{A} \times \mathcal{W}$, where $\mathcal{W} \equiv [\underline{m}_0 + P\underline{l}_0, \bar{m}_0 + P\bar{l}_0]$.

We impose minimal restrictions on the joint distribution: the minimum farming ability $\underline{a}$ is strictly positive, and strictly smaller than the highest ability $\bar{a}$ in the population. Denoting the aggregate per capita date 0 endowment of land and the M-good in the economy by λ and μ respectively, we assume $\lambda \in (0, 1)$ so there is not enough land in the economy to allow everyone to farm full-time.

Moreover to ensure that all agents will have sufficient income at date 1 in both states to be able to cover subsistence requirements, we assume:

$$\frac{s}{\lambda^{1-\alpha} A_d \underline{a}} < \min\{\gamma, \frac{1-\gamma}{1+\gamma(\mu-\lambda)}\} \tag{4}$$

In order to ensure interior land size choices for every agent we additionally assume

$$\bar{a} < \frac{A_d \lambda^{1-\alpha} \underline{a} - s}{k A_d} \tag{5}$$

where $\bar{a}$ is the maximum ability in the population.

Later in order to focus on the security value of land in isolation from wealth constraints, we shall impose a restriction that $\underline{m}_0 \geq 1$.

In the analysis that follows we shall examine patterns of land demand for all possible ability-wealth types in the set $\mathcal{A} \times \mathcal{W}$, irrespective of whether they actually exist in the population. Market clearing conditions will require per capita demand and supply to be equated, where these averages are computed using the actual type distribution. Hence the analysis applies both to a continuum economy with an atomless type distribution as well as an economy with a finite number of agents and a finite type distribution.

Note some features of this simple model. It abstracts from working capital requirements.

Land rights are the same as cultivation rights, so there is no distinction between landownership and tenancy. One can interpret the land market either as sale of land or of renting out land to a tenant on a fixed rent contract. The absence of state-contingent contracts rules out the possibility of sharecropping tenancy, for various un-modeled reasons such as high enforcement costs or moral hazard. The results hopefully will continue to apply as long as the extent of risk-sharing that can be achieved via sharecropping is limited. Moreover, as farming involves self-cultivation, there are no agricultural labour markets. These simplifying assumptions enable us to focus on the implications of weather risk, a covariate macro shock for the economy. In the absence of insurance markets, land cultivation rights represent a bundle of two attributes: the opportunity to produce crops which generate an income, as well as partial insurance against weather shocks. Hence land will be valued both for its productivity and insurance role.

2.2 Date 1 Spot Market Equilibrium

We solve backwards, starting with date 1 commodity spot prices in each state w, conditional on an allocation of land and wealth across agents resulting from an equilibrium of the land market at date 0.

Lemma 1 *Let $l(a, W_0)$ denote the land size held by an agent of ability a and initial wealth W_0 following an equilibrium of the date 0 land market, and let $q(a, W_0) \equiv al(a, W_0)^{1-\alpha}$. Then the date 1 spot price in state w will be*

$$p_w = \frac{k}{A_w E[q(a, W_0)] - s} \tag{6}$$

where k denotes $\frac{\gamma}{1-\gamma}[1 - \lambda + \mu]$ and $E[q(a, W_0)]$ denotes expected value of $q(a, W_0)$ in the economy over the joint distribution of (a, W_0).

In this equilibrium: (a) every agent will have enough income in both states to cover the cost of subsistence, and (b) food prices and farm revenues are higher in the drought state ($p_d > p_n$, $p_d.A_d > p_n.A_n$), while the date 1 indirect utility of every agent is lower.

Proof of Lemma 1: Clearing of the land market implies per capita labor supply in the food sector is λ, and in the M-sector output produced equals $1 - \lambda$. Hence total per capita supply of the M-good on the spot market will be the sum of production and inventory $1 - \lambda + \mu$. The supply of food in state w will be $A_w E[q(a, W_0)]$. Hence the per capita household budget in the economy will be $p_w A_w E[q(a, W_0)] + 1 - \lambda + \mu$. Assuming for the time being that every agent has enough income to cover subsistence needs, the aggregate demand for the M-good in state w will be $(1-\gamma)[p_w(A_w E[q(a,W_0)]-s)+1-\lambda+\mu]$. Therefore the equilibrium price of food in state w is obtained by the condition that the market for the M-good clears:

$$1-\lambda+\mu=(1-\gamma)[p_w(A_w E[q(a,W_0)]-s)+1-\lambda+\mu] \tag{7}$$

which implies (6).

For this to be a spot market equilibrium, every agent must have enough income to cover subsistence needs. Since $E[q(a,W_0)]$ is bounded below by $\lambda^{1-\alpha}\underline{a}$, (4) ensures that every agent can always afford subsistence consumption. This is because: (a) (4) implies $p_w.s<1, w=d,n$, and (b) an agent with ability a choosing l earns an income of at least ap_wA_wl+1-l which exceeds p_ws since (4) also implies $aA_w>s$.

Next, note that (6) implies that p_w and $p_wA_w=\frac{kA_w}{A_wE[q]-s}$ are both decreasing in A_w, implying that $p_d>p_n$ and $p_d.A_d>p_n.A_n$. The indirect utility of an agent type (a,W_0) is

$$\frac{p_w.A_w.q(a,W_0)-p_ws-(1+P)l(a,W_0)+1+W_0}{p_w^{\gamma}} \tag{8}$$

This is lower in the drought state for two reasons. First, the cost of meeting subsistence requirements p_ws is higher in state d. Second, (4) implies $\gamma E[q]A_w>s$, which in turn implies $(p_w)^{1-\gamma}A_w=[k]^{1-\gamma}\frac{A_w}{(A_wE[q]-s)^{1-\gamma}}$ is increasing in A_w. Hence the higher income from food sales is insufficient to compensate for higher cost of consuming food: $\frac{p_w.A_w}{p_w^{\gamma}}=(p_w)^{1-\gamma}A_w$ is lower in state $w=d$. ■

Food prices are higher in drought owing to the shortage of food supplies. As food is a necessity, its demand is price-inelastic, so crop revenues are higher in droughts. Hence land ownership provides some insurance against the higher cost of living in a drought. The insurance is partial: all agents are worse off as the higher crop incomes are insufficient to compensate for the higher cost of living when there is a drought.

2.3 Date 0 Land Market Equilibrium

At date 0, all agents anticipate date 1 spot prices correctly, and take the prevailing land price P as given. Given land price of P and spot food price p_w in state w, an agent of ability a and wealth W_0 selects land area l to maximize $\sum_w f_wV(\frac{Y_w-p_ws}{p_w^{\gamma}})$ subject to

$$0\le l\le \min\{1,\frac{W_0}{P}\} \tag{9}$$

where recall that

$$Y_w=ap_wA_wl^{1-\alpha}-(1+P)l+1+W_0 \tag{10}$$

Note that this is a strictly concave optimization problem. So first-order conditions are necessary and sufficient to characterize optimal land demands, and comparative static results with respect to parameter values can be obtained by differentiating these conditions.

In this formulation, agents are subject to a borrowing constraint, imposing a wealth constraint on landownership: $Pl \leq W_0$. An insurance market friction therefore co-exists with a borrowing constraint, which additionally complicates the model (but circumvents the need to incorporate a credit market). It is conceptually useful to understand the distortions created by the insurance friction, in abstraction from effects of borrowing constraints which have been widely studied in the literature. In order to do so, our main result below (Proposition 3) will impose an additional restriction on the distribution of date 0 wealth which will ensure the wealth constraint will not bite. Nevertheless we include it for the time being.

The resulting land demand function is

$$\min\{l_d(a, W_0; P, p_d, p_n), \frac{W_0}{P}\} \tag{11}$$

where

$$l_d(a, W_0; P, p_d, p_n) = \left[\frac{(1-\alpha)a\sum_w \phi_w p_w A_w}{1+P}\right]^{\frac{1}{\alpha}} \tag{12}$$

$$\phi_w \equiv \frac{\frac{f_w V_w}{p_w^\gamma}}{\sum_{w'} \frac{f_{w'} V_{w'}}{p_{w'}^\gamma}} \tag{13}$$

and V_w denotes marginal utility $V'(\frac{Y_w - p_w s}{p_w^\gamma})$ in state w. By construction $\phi_d + \phi_n = 1$, so ϕ_w represents a set of 'welfare' weights with respect to which expectation is taken over the value of the marginal product of land and equated to its marginal cost $1 + P$ to determine land demand l_d when the wealth constraint does not bind. Since $E[al(a, W_0)^{1-\alpha}]$ is bounded below by $\underline{a}\lambda^{1-\alpha}$, (5) ensures that $l_d(a, W_0; P, p_d, p_n)$ is in the interior of $(0, 1)$.[10]

Finally, an **equilibrium** of the economy is a date 0 land price P, a set of date 1 spot prices p_d, p_n (described in Lemma 1) and a land allocation such that the land markets clears, where demand for land is described by the solution to the optimization problem given in the preceding paragraph. This is a special case of a Radner (1972) equilibrium of an economy with incomplete markets.

Misallocation is defined relative to a first-best benchmark, where agents can purchase Arrow securities and are not subject to wealth constraints. There land demanded by an

[10] (5) implies ap_dA_d is smaller than 1 for all a. Since the weights ϕ_w add up to 1, $(1-\alpha)a\sum_w \phi_w p_w A_w$ is bounded above by ap_dA_d. Therefore $l_d > 0$ is smaller than one.

agent of type (a, W_0) equals

$$l_d^*(a; P, p_d, p_n) = \left[\frac{(1-\alpha)a\sum_w \psi_w p_w A_w}{1+P}\right]^{\frac{1}{\alpha}} \tag{14}$$

and first-best welfare weights

$$\psi_w \equiv \frac{\frac{f_w}{p_w^\gamma}}{\sum_{w'} \frac{f_{w'}}{p_{w'}^\gamma}} \tag{15}$$

which are independent of the agent's wealth or ability.

In the second-best setting, indirect utility is lower in the drought state, implying $V_d > V_n$. Hence $\phi_d > \psi_d$, i.e., a higher welfare weight is assigned to the drought state. The unconstrained land demand function can be expressed as

$$l_d(a, W_0; P, p_d, p_n) = \left[\frac{(1-\alpha)\{a\sum_w \psi_w p_w A_w + (\phi_d - \psi_d)(p_d A_d - p_n A_n)\}}{1+P}\right]^{\frac{1}{\alpha}} \tag{16}$$

which implies land demand is higher in the second-best setting (unless the wealth constraint binds):

$$l_d(a, W_0; P, p_d, p_n) > l_d^*(a; P, p_d, p_n) \tag{17}$$

for any W_0.

The divergence between second-best and first-best land demand provides a convenient measure of the security value of land which causes all agents to demand more land owing to its role in providing (partial) insurance. It also follows that the second-best price of land will exceed the first-best price in the land market equilibrium, again provided wealth constraints do not bind.

A measure of the resulting land 'misallocation' in the second-best setting is the following. Taking the agent with some fixed ability $\hat{a}$ and wealth $\hat{W}_0$ as a benchmark, consider the ratio of land allocated to an agent of ability a and wealth W_0 relative to the benchmark agent, where neither is wealth constrained. In the first-best setting this ratio equals

$$\left[\frac{a}{\hat{a}}\right]^{\frac{1}{\alpha}} \tag{18}$$

which is independent of prices. In the second-best setting it is given by

$$\frac{l_d(a, W_0; P, p_d, p_n)}{l_d(\hat{a}, \hat{W}_0; P, p_d, p_n)} = \left[\frac{a + (p_d A_d - p_n A_n)[\sum_w \psi_w p_w A_w]^{-1}[\phi_d(a, W_0) - \psi_d]}{\hat{a} + (p_d A_d - p_n A_n)[\sum_w \psi_w p_w A_w]^{-1}[\phi_d(\hat{a}, \hat{W}_0) - \psi_d]}\right]^{\frac{1}{\alpha}} \tag{19}$$

which is independent of the second-best land price but depends on second-best spot prices. The pattern of variation of ϕ_d across agents of varying ability and wealth represents the corresponding distortion in land allocation resulting from the security value of land. So ϕ_d is a convenient proxy for the security value of land.

Our main question of interest is the pattern of misallocation and how it depends on risk attitudes.

Proposition 2 *If ARA is non-decreasing, ϕ_d is increasing both in ability and wealth.*

Proof: ϕ_d is a monotone increasing transformation of $\frac{V_d}{V_n}$, so it suffices to examine how the latter varies with ability and wealth. Consider first the sign of the wealth effect:

$$\text{sign}\{\frac{\partial}{\partial W_0}[\frac{V_d}{V_n}]\} = \text{sign}\{V_n V_d[\frac{ARA_n}{p_n^\gamma} - \frac{ARA_d}{p_d^\gamma}]\} \tag{20}$$

where ARA_w denotes absolute risk aversion evaluated at $\frac{Y_w - p_w s}{p_w^\gamma}, w = d, n$. Since $\frac{Y_w - p_w s}{p_w^\gamma}$ and $p_w^{-\gamma}$ are both higher at $w = n$, non-decreasing ARA implies the wealth effect is positive.

Next consider the sign of the ability effect:

$$\text{sign}\{\frac{\partial}{\partial a}[\frac{V_d}{V_n}]\} = \text{sign}\{V_n V_d[ARA_n \frac{p_n A_n}{p_n^\gamma} - ARA_d \frac{p_d A_d}{p_d^\gamma}]\} \tag{21}$$

This is positive because $\frac{p_w A_w}{p_w^\gamma}$ is increasing in A_w. ■

So if wealthier agents have higher or the same ARA as poorer agents, the insurance frictions skew the land distribution in their favor, as they place a higher value on the insurance benefits of land. The distribution is also skewed in favor of more able agents, for two reasons. One is that more able agents end up earning more from a given amount of land, thereby generating a positive wealth effect. This is compounded by the higher 'real' return to ability $\frac{p_w A_w}{p_w^\gamma}$ in the normal state relative to the drought state, which accentuates the discrepancy between farm incomes in the two states and thereby raises the value of insurance.

However, non-decreasing ARA is inconsistent with what is considered a stylized fact in finance that wealthier agents invest less in safer assets. We therefore turn to contexts where ARA is decreasing, and in particular to the most commonly employed specification of constant RRA.[11] If ARA is decreasing, this would tend to impart a misallocation bias in favor of poorer agents who are more risk averse (as measured by ARA). However, expression

[11] Indeed some authors in finance (see e.g., Calvet and Sodini 2014 and Morin and Suarez 1983 believe a stronger hypothesis of decreasing RRA may be more appropriate, to explain why wealthier agents tend to invest a smaller share of their portfolios in safer assets. We conjecture that the results we obtain below with CRRA will be accentuated further if RRA is decreasing.

(20) shows that this does not suffice to generate negative wealth effects, since there is also a reverse effect arising from the higher 'real' value of increased wealth in the normal state (owing to a lower cost of living) which increases the value of insurance (by increasing the discrepancy in consumption between the two states). So it is not immediately obvious what the net result of these two contrasting effects will be.

To focus on this question, it is analytically simpler and more convenient to focus on the effect of insurance frictions in isolation from the wealth constraint which reflects capital market frictions. We show below that a sufficient condition for the wealth constraint to not bind for any agent is the following:

$$\underline{m}_0 \geq 1 \tag{22}$$

where $\underline{m}_0$ denotes the lowest holding of the date 0 M-good stock across all agents in the economy.

Proposition 3 *Suppose RRA is constant.*

(i) The security value of land ϕ_d is decreasing in ability, given any initial wealth W_0.

(ii) If (22) holds, the following properties are satisfied in any equilibrium allocation:

(a) Land demanded by any agent is not wealth constrained.

(b) Land owned by an agent is increasing in ability, holding wealth constant.

(c) There exists an intermediate ability level $a_m \in (\underline{a}, \bar{a})$ such that ϕ_d and land owned by agents with ability a_m is independent of W_0, and for those with ability below (resp. above) a_m is decreasing (resp. increasing) in W_0.

Proof: (i) With a constant RRA of ρ, the ability effect

$$\begin{aligned} \text{sign}\{\frac{\partial}{\partial a}[\frac{V_d}{V_n}]\} &= \text{sign}\{V_n V_d[\rho\frac{p_n A_n}{Y_n - p_n s} - \rho\frac{p_d A_d}{Y_d - p_d s}]\} \\ &= \text{sign}[(p_n A_n - p_d A_d)(1 + W_0 - l(1+P)) + p_n p_d(A_d - A_n)s] \end{aligned} \tag{23}$$

is negative if ϕ_d is evaluated at any $l \leq 1$ where the wealth constraint $W_0 \geq Pl$ holds.

(ii) Start with (a). Condition (22) combined with (5) implies that for any agent:

$$W_0 \geq \underline{m}_0 \geq 1 \geq \frac{\bar{a}kA_d}{A_d\lambda^{1-\alpha}\underline{a} - s} \geq \bar{a}p_d A_d \geq P \geq Pl \tag{24}$$

for any $l \leq 1$. The third inequality (from left to right) is a restatement of (5). The fourth inequality holds because $\lambda^\alpha \underline{a}$ is a lower bound to per capita food output in the economy. The fifth inequality holds since $\bar{a}p_dA_d - 1$ (and hence $\bar{a}p_dA_d$) is an upper bound on the willingness to pay for land for any agent, and therefore also an upper bound on the equilibrium land price P.

This implies that the land allocated to each agent in any date 0 equilibrium must equal its unconstrained land demand at equilibrium prices.

Next we prove (b). With CRRA denoted by ρ:

$$\frac{\partial V(\frac{Y_w - p_w s}{p_w^\gamma})}{\partial l} = \left[\frac{(1-\alpha)ap_wA_wl^{-\alpha} - (1+P)}{p_w^\gamma}\right]\left[\frac{ap_wA_wl^{1-\alpha} + S_w}{p_w^\gamma}\right]^{-\rho} \tag{25}$$

where S_w denotes $W_0 + 1 - (1+P)l - p_w s$. Since the expected utility of the agent equals $\sum_w f_w V(\frac{Y_w - p_w s}{p_w^\gamma})$, it suffices to show that

$$\frac{(1-\alpha)ap_wA_wl^{-\alpha} - (1+P)}{[ap_wA_wl^{1-\alpha} + S_w]^\rho} \tag{26}$$

is increasing in a (holding l, W_0, P, p_s, p_d fixed) for both $w = d, n$, upon using a standard monotone comparative static argument.

Observe that $S_w > 0$ for $w = d, n$ for the following reason. (24) implies $W_0 \geq \bar{a}p_dA_d$. Since $p_n s < p_d s < 1$, we deduce $W_0 - p_n s > W_0 - p_d s \geq \bar{a}p_dA_d - p_d s > \bar{a}p_dA_d - 1 > P$. Hence $S_w = W_0 - p_w s - Pl + (1-l) \geq (W_0 - p_w s) - P + (1-l) > 0$ for $w = d, n$.

This implies (26) is increasing in a when $\rho \geq 1$. This is because (26) equals

$$\frac{(1-\alpha)p_wA_wl^{-\alpha} - \frac{(1+P)}{a}}{[a^{1-\rho}p_wA_wl^{1-\alpha} + a^{-\rho}S_w]^\rho} \tag{27}$$

and the numerator is increasing in a while the denominator is decreasing in a if $\rho \geq 1$ given $S_w > 0$.

(26) can alternatively be rewritten as

$$(1-\alpha)p_wA_wl^{-\alpha}\frac{a}{[ap_wA_wl^{1-\alpha} + S_w]^\rho} - \frac{(1+P)}{[ap_wA_wl^{1-\alpha} + S_w]^\rho} \tag{28}$$

The second term is decreasing in a for any $\rho > 0$. The first term is increasing in a for any $\rho \in (0,1)$, because this requires $\frac{1}{a} > \rho\frac{p_wA_wl^{1-\alpha}}{ap_wA_wl^{1-\alpha}+S_w}$, which is satisfied as $\rho < 1$ and $S_w > 0$. So (26) is also increasing in a when $\rho \in (0,1)$. This concludes the proof of (b).

Next, we prove (c). Observe that given any set of prices P, p_s, p_d, the function $Q(a, W_0; P, p_s, p_d) \equiv al_d(a, W_0; P, p_d, p_s)^{1-\alpha}$ is a continuous function from the compact type space $\mathcal{A} \times \mathcal{W}$ into the real line. Hence there exists a type $(a_m, W_{0m}) \in \mathcal{A} \times \mathcal{W}$ such that $Q(a_m, W_{0m}; P, p_s, p_d) =$

$E[Q(\tilde{a}, \tilde{W}_0; P, p_s, p_d)]$ the expected value of Q in the economy. This implies that in either state w, type (a_m, W_{0m}) is a 'representative' agent who produces the same food as the economy wide-average.

In what follows we suppress prices P, p_s, p_d to simplify the notation.

Claim 1: $l_d(a_m, W_0) = l_d(a_m, W_{0m})$ for all $W_0 \in \mathcal{W}$.

Starting with the representative agent (a_m, W_{om}), consider an agent with the same ability but one whose wealth W_0 is slightly higher than W_{0m}. From (16) it follows that the resulting change in land demand depends on how ϕ_d changes locally. Given CRRA, this in turn depends on whether 'nominal' discretionary spending $Y_w - p_w s$ is lower (resp. higher) in state n than state d. The proportional increase in discretionary income resulting from the wealth increase would then be higher (resp. smaller) in state n. The ratio of indirect utility in the drought state to that in the normal state $[\{\frac{Y_d - p_d s}{Y_n - p_n s}\}\frac{p_d^\gamma}{p_n^\gamma}]^\rho$ would fall (resp. rise): $\frac{V_d}{V_n}$ and thus ϕ_d and land demand would rise (resp. fall). If discretionary income is the same in both states, land demand would remain the same, and so would food output produced by the agent in a market clearing equilibrium.

Now for an agent with arbitrary ability a and wealth W_0:

$$\begin{aligned} Y_w - p_w s &= p_w[aA_w\{l(a, W_0)\}^{1-\alpha} - s] + W_0 - (1+P)l(a, W_0) \\ &= k[\frac{A_w q(a, W_0) - s}{A_w E[q(\tilde{a}, \tilde{W}_0)] - s}] + W_0 - (1+P)l(a, W_0) \end{aligned} \tag{29}$$

Hence for such an agent $Y_w - p_w s$ is higher (resp. lower) in state n if $\frac{A_w q(a,W_0) - s}{A_w E[q(\tilde{a}, \tilde{W}_0)] - s}$ is rising (resp. falling) in A_w.

But by definition, the representative agent with ability a_m and wealth W_{0m} produces the same output as the average per capita food output: $q(a, W_0) = E[q(\tilde{a}, \tilde{W}_0)]$. For such an agent, $\frac{A_w q(a,W_0) - s}{A_w E[q(\tilde{a}, \tilde{W}_0)] - s}$ equals 1 irrespective of A_w. Therefore, the local wealth effect for such an agent is exactly zero.

This in turn implies that $\phi_d(a_m, W_0)$ is locally constant with respect to W_0 in a neighborhood of W_{0m}. Hence the land demand is locally invariant to wealth changes around W_0, which in turn implies that the food produced by an agent with ability a_m and wealth in a local neighborhood of W_0 is also constant.

The following argument shows why the same result holds globally for all W_0, not just at W_{0m}. Observe that there exists a solution to the first order conditions (12, 13) with this property, since $\phi_d(a_m, W_0)$ is independent of W_0 if $l_d(a_m, W_0)$ is independent of W_0, and the converse is also true. To see this, observe that if we set $\phi_d(a_m, W_0) = \phi_d(a_m, W_{0m})$ for all W_0, it follows from (12) that the corresponding land allocation $l_d(a_m, W_0) = l_d(a_m, W_{0m})$

for all W_0. This in combination with (13) implies that all agents of ability a_m produce the same food output, consistent with the assumption that $\phi_d(a_m, W_0) = \phi_d(a_m, W_{0m})$ for all W_0. Since the agent's optimization problem is strictly concave (at any given set of prices), there is a unique solution to the first order conditions. Hence the constructed function is the optimal solution for every agent with $a = a_m$, irrespective of W_0, concluding the proof of Claim 1.

Claim 2: Wealth effects are negative (resp. positive) for any ability below (resp. above) a_m.

Fix any wealth level W_0. Part (b) implies land and thus food output produced by any agent with ability a below (resp, above) a_m is smaller (resp. larger) than the food output of type (a_m, W_0). Claim 1 implies the latter output equals the economy-wide average food output. From the reasoning employed (see (29)) in proving Claim 1, discretionary income for agents with ability smaller (resp. higher) than a_m is higher (resp. smaller) in state n. An increase in wealth therefore results in a smaller (resp. larger) proportional effect on discretionary income and indirect utility in the normal state. Hence $\phi_d(a, W_0)$ must be decreasing (resp. increasing) in W_0 if a is lower (resp. higher) than a_m. This concludes the proof of Claim 2.

To complete the proof of (c), it remains to show that a_m cannot equal either $\underline{a}$ or $\bar{a}$. Suppose $a_m = \underline{a}$. Then $l_d(a, W_0)$ and hence $q(a, W_0)$ achieves a minimum at ability $\underline{a}$ and any wealth level W_{0m} over the entire type space $\mathcal{A} \times \mathcal{W}$, because at any $a > a_m$ and any $W_0 \in \mathcal{W}$ we have $l_d(a, W_0) \geq l_d(a, \underline{W}_0) > l_d(\underline{a}, \underline{W}_0) = l_d(\underline{a}, W_{0m})$. Since the ability distribution is non-degenerate, the expected value of $q(a, W_0)$ over the type space must be strictly higher than $q(\underline{a}, W_{0m})$, a contradiction. A similar argument in reverse rules out the possibility that $a_m = \bar{a}$. ■

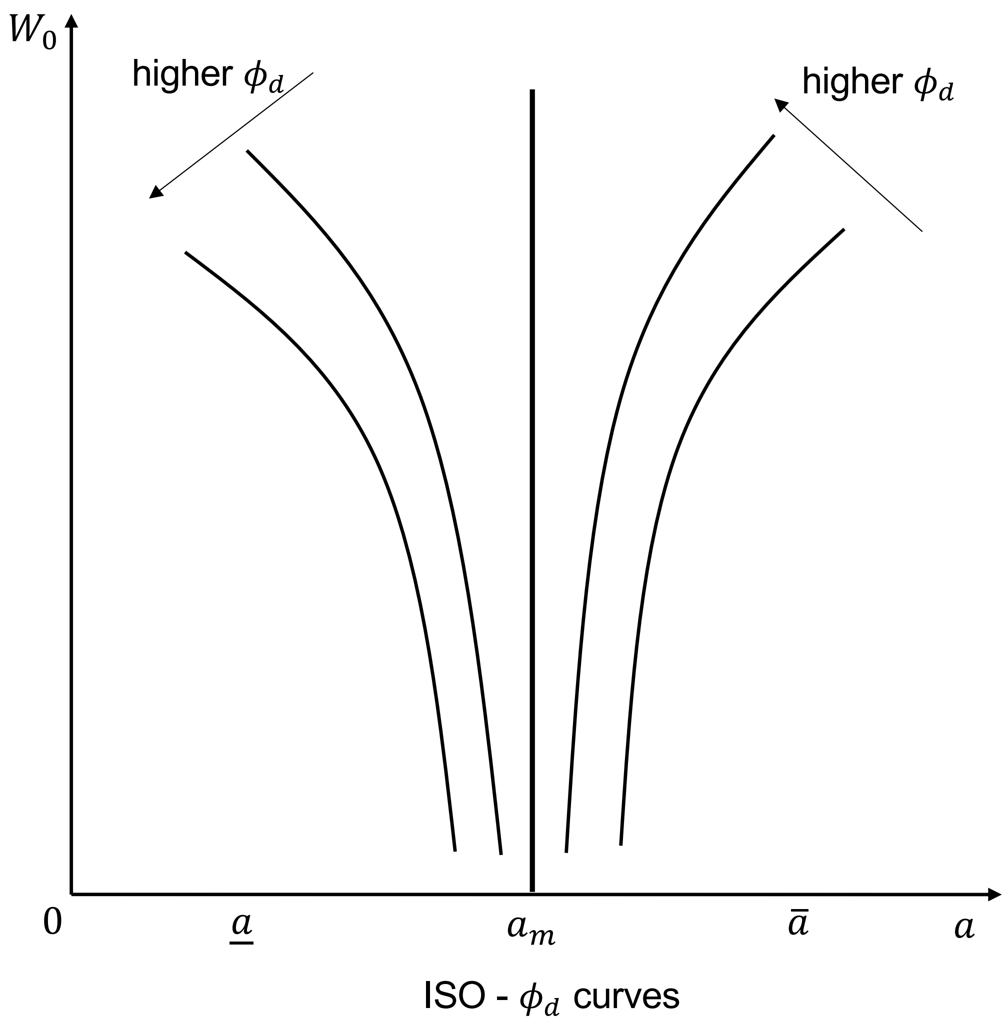
With CRRA we obtain rather striking implications of the insurance market friction. Given any wealth category W_0, lower ability agents have a greater security value of land, biasing the allocation in their favor. Intuitively this is because higher ability agents are exposed to less drought risk owing to a higher marginal return to ability in farming in the drought state for any given land size (as $p_d A_d > p_n A_n$). This is distinct from the case of NDARA, or most models of credit market frictions.

The pattern of wealth effects are also notable, as they differ between low and high ability agents (relative to an intermediate threshold ability a_m).[12] Wealth effects on ϕ_d are negative

[12] Note that there may be no actual type in the economy with ability exactly equal to a_m — all we have shown is that there exists such an ability type in the support of the type space $\mathcal{A}$. For instance if there are just two ability types $\underline{a}, \bar{a}$ in the population, there is actually no one with the ability a_m.

among low ability ($a < a_m$) agents and positive for high ability ($a > a_m$) agents. We provide some intuition for this later in this Section. Figure 1 illustrates the resulting iso-ϕ_d curves in $a - W_0$ space. It shows the bias in favor of low ability, low wealth agents. The security value is highest for agents with the lowest ability and wealth in the population. A low ability agent has a higher security value than any high ability agent, irrespective of their respective wealth levels (because $\phi_d(a_1, W_{01}) > \phi_d(a_m, W_{0m}) > \phi_d(a_2, W_{02})$ for any $a_1 < a_m < a_2$ and arbitrary wealth levels W_{01}, W_{0m}, W_{02}[13]). It follows that average land productivity in the economy is lower in the second-best compared with the first-best, implying a greater food scarcity in the economy which raises food prices and further aggravates its dispersion between the drought and normal states.

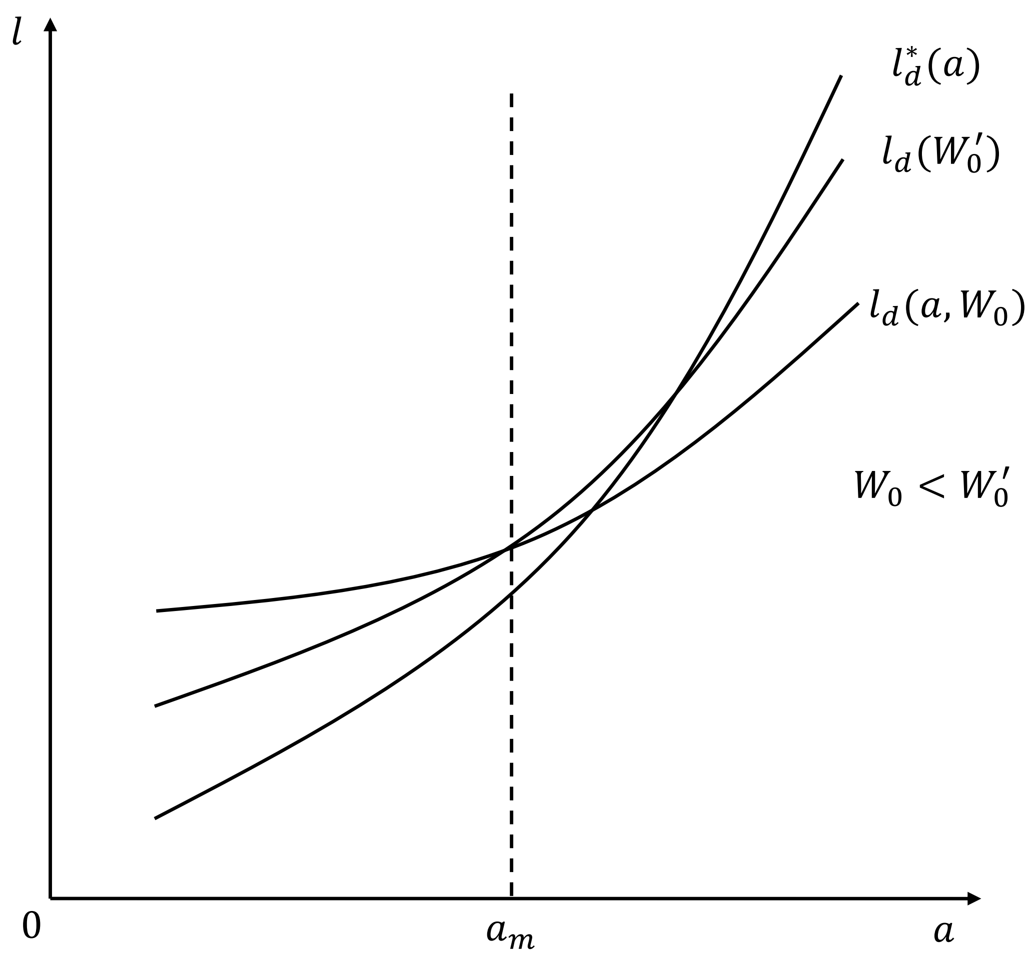
FIGURE 1: **Ability and Wealth Effects on Land Security Value**



[13]This is because $\phi_d(a_1, W_{01}) > \phi_d(a_m, W_{01}) = \phi_d(a_m, W_{02}) > \phi_d(a_2, W_{02})$: see Figure 1.

Of course a higher security value of land for less able agents does not necessarily imply they end up with a higher land allocation. This is because of a tension between the intrinsic positive effect of higher ability on productivity and the negative effect on security value which move in opposite directions (see equation (16)). Part (ii) of Proposition 3 shows that the former effect always dominates, so higher ability agents are always assigned more land (controlling for initial wealth W_0). Rather, a higher security value for low ability agents implies the land allocation is distorted more in favor of low ability agents (see (19)). Figure 2 illustrates how the second best land allocation compares with the first-best for two different wealth categories. Land allocated to the highest ability agents must be lower compared to the first-best, both due to a higher land price in the second-best economy and these agents having the lowest value for security in the population.

FIGURE 2: **Second-Best versus First-Best Land Allocation**



Compared with the first-best, it follows that the second-best economy will be characterized by lower per capita agricultural productivity, higher average relative food price as well as volatility.[14] This suggests that the model might provide predictions consistent with cross-country patterns, but confirming this will require comparative static analyses of the second-best equilibrium with parameters such as average wealth or availability of state-provided partial drought relief.

We conclude with an intuitive explanation of the pattern of wealth effects under CRRA. The crux of the argument is the following. With CRRA, the effect of raising wealth W_0 by one unit on ϕ_d depends on how it affects the ratio of indirect utility in the two states. Under CRRA, this ratio equals $[\frac{Y_d - p_d s}{Y_n - p_n s}]^\rho [\frac{p_n^\gamma}{p_d^\gamma}]^\rho$. Since each agent is a price-taker, this is determined by the effect of raising W_0 on the ratio of discretionary expenditures $\frac{Y_d - p_d s}{Y_n - p_n s}$. If this ratio rises, the agent is better insured and places a lower value on security. Since $Y_w \equiv a p_w A_w l(a, W_0)^{1-\alpha} + W_0 + (1 - l(a, W_0)) - (1 + P) l(a, W_0)$, a unit increase in W_0 raises Y_w by the same one unit in both states. So what matters is the proportional change in discretionary spending in the two states, which depends on which state features a higher base level of discretionary spending. In other words it depends on

$$(Y_d - p_d s) - (Y_n - p_n s) = (p_d A_d - p_n A_n) q(a, W_0) - (p_d - p_n) s \tag{30}$$

where recall $q(a, W_0) \equiv a l(a, W_0)^{1-\alpha}$ is increasing in a. The first term in (30) is positive (reflecting the higher crop income in the drought state), while the second term is negative (reflecting the higher cost of subsistence during drought). The former effect is larger, the higher the agent's ability. So discretionary spending is higher in the drought state if and only if the first effect dominates, i.e., ability is above a threshold $a_m(W_0)$ where the two effects offset each other perfectly, i.e.,

$$q(a_m(W_0), W_0) = \frac{p_d - p_n}{p_d A_d - p_n A_n} s \tag{31}$$

For low ability agents, discretionary spending is lower in the drought state at the baseline, owing to the predominance of the higher cost of subsistence over higher crop incomes. A unit increase in wealth raises spending equally in both states, thus increasing spending in the drought state by a higher proportion, i.e., providing better insurance. This lowers the security value of land for low ability agents. A converse argument explains why it rises for high ability agents.

Of course this does not explain why the ability threshold a_m is independent of W_0 or why it lies in the interior of the ability range. Proving these properties requires more detailed arguments provided in the proof.

[14]Expression (6) shows a lower average food productivity raises spot food prices in each state, as well as the dispersion between drought and normal states.

2.4 Constrained Inefficiency and Welfare Effects of Policy Interventions

With incomplete markets, competitive equilibria rarely achieve first-best Pareto optimality. Accordingly the theoretical literature has explored properties of constrained Pareto optimality (CPO) in which a social planner is limited to reallocating via the markets that actually exist in the economy. Geakanokoplos and Polemarchakis 1986 study a more general class of models than considered in this paper, where Arrow-Debreu security markets are missing, agents trade assets at an initial *ex ante* date, followed by spot market trades in physical commodities at a second date after an *ex ante* uncertain state of nature is revealed. Under some weak assumptions (pertaining to the number of households relative to the number of physical goods and states of nature) they show that competitive equilibria generically fail to be CPO in the following sense: starting with the asset allocation achieved in the equilibrium at the end of the first date, there exists a reallocation of these assets such that all agents are better off (in terms of *ex ante* welfare) in the resulting spot market equilibrium at the second date. The economic intuition for this result (also elaborated by Stiglitz 1982 in a parallel paper) is that asset redistributions result in changes in spot market prices at the second date which affect the welfare of all agents, representing an economy-wide pecuniary externality. Barring exceptional cases (such as homothetic preferences, lack of income effects in spot market demand, or Pareto optimality of autarkic allocations), it is possible for a social planner to construct an asset redistribution that affects spot market prices in way that benefits all agents *ex ante*. However, in order to construct such a redistribution, the planner may need to be fully informed about technology and the distribution of tastes and endowments in the entire economy.

To provide a better sense of the nature of inefficiency and the required informational requirements for a social planner, we illustrate the argument for a special case of our model with two ability types $\underline{a} < \bar{a}$, where all agents own the same endowments of land (λ) and M-good stock (μ) at the beginning of date 0. Let $\pi \in (0, 1)$ denote the fraction of low ability types. Consider an equilibrium allocation with land sizes $\underline{l}, \bar{l}$ for the two ability types, land price P and spot food prices $p_w = \frac{k}{A_w E[q]-s}$ where $E[q] = \pi \underline{a}\underline{l}^{1-\alpha} + (1-\pi)\bar{a}\bar{l}^{1-\alpha}$ and $\pi\underline{l} + (1-\pi)\bar{l} = \lambda$. Land misallocation implies a lower marginal product of low ability agents:

$$\underline{a}\underline{l}^{-\alpha} < \bar{a}\bar{l}^{-\alpha} \tag{32}$$

Stocks of the M-good held by the two types at the end of date 0 are $\mu + P(\lambda - \underline{l})$ and $\mu + P(\lambda - \bar{l})$ respectively.

Consider the following perturbation of asset stocks at the end of date 0, with scale parametrized by $\epsilon > 0$. The land owned by the low ability agent falls by ϵ, accompanied by an increase in $\frac{\pi}{1-\pi}\epsilon$ in land owned by the high ability type. The land reallocation is accompanied by corresponding reallocation of M-good stocks where the change in land sizes are valued at the equilibrium price P, i.e., each low ability type receives an additional $P\epsilon$ units of the M-good, while the high ability type gives up $P\frac{\pi}{1-\pi}\epsilon$. Hence the perturbation corresponds to a variation on the equilibrium allocation with the low (resp.) ability types acquiring a little less (more) land on the date 0 land market at the equilibrium price P.

At date 1 the spot price of food in state w changes to $p_w(\epsilon) = \frac{k}{A_w E[q(\epsilon)]-s}$ where

$$E[q(\epsilon)] = \pi\underline{a}(\underline{l} - \epsilon)^{1-\alpha} + (1-\pi)\bar{a}(\bar{l} + \frac{\pi}{1-\pi}\epsilon)^{1-\alpha} \tag{33}$$

This implies

$$\lim_{\epsilon \to 0} \frac{\partial p_w(\epsilon)}{\partial \epsilon} = \frac{k}{[A_w E[q] - s]^2}(1-\alpha)\pi[\underline{a}\underline{l}^{-\alpha} - \bar{a}\bar{l}^{-\alpha}] \tag{34}$$

which is strictly negative owing to (32). The increase in average food productivity lowers the spot price of food in both states. This is the key pecuniary externality which benefits both types of agents.

On the other hand the direct impact of changes in land sizes lowers the expected utility of both types (holding prices fixed), since they deviate from their (privately) optimal land demands. But for ϵ sufficiently small, this loss is second order, and will be outweighed by the first-order gain from lower food prices — resulting in a Pareto improvement. To see this, consider the ex ante welfare of the low ability type $\sum_w f_w V(\underline{u}_w)$ where

$$\underline{u}_w \equiv p_w(\epsilon)^{-\gamma}[\underline{a}p_w(\epsilon)A_w(\underline{l} - \epsilon)^{1-\alpha} - p_w(\epsilon)s + 1 - \underline{l} + \epsilon + P(\lambda - \underline{l} + \epsilon)] \tag{35}$$

Hence

$$\begin{aligned} \frac{\partial}{\partial \epsilon}[\sum_w f_w V(\underline{u}_w)] &= \sum_w f_w V'(\underline{u}_w)[p_w(\epsilon)]^{-\gamma}\{(1+P) - (1-\alpha)\underline{a}p_w(\epsilon)A_w(\underline{l} - \epsilon)^{-\alpha}] \\ &\quad + \sum_w f_w V'(\underline{u}_w)\frac{\partial \underline{u}_w}{\partial p_w}\frac{\partial p_w(\epsilon)}{\partial \epsilon} \end{aligned} \tag{36}$$

The first term on the right-hand-side converges to zero as $\epsilon \to 0$ by virtue of the first order condition which applies at $\underline{l}$ for the low ability type in equilibrium. The second term converges to a strictly positive number owing to (34). In the decentralized equilibrium each agent ignores the effect of its landownership decision on spot prices (the second term on the right-hand-side of (36)), thereby creating room for a coordinated deviation in land allocation that benefits everyone. Note also that the welfare improvement is essentially driven by a reduction in productive misallocation.

In this example, the only information required for a social planner to implement the Pareto improving reallocation is not particularly demanding. If the planner can observe land transactions (or farm sizes), it can implement the Pareto improving reallocation with a non-linear (size-dependent) tax-subsidy based on these. However, matters are more complicated if we introduce wealth heterogeneity, since farm size will then no longer reveal the agents farming ability. One can extend the preceding policy variation with one that reallocates land from low to high ability agents, but would require the planner to observe the wealth of agents and condition the corresponding tax-subsidy scheme on it.

3 Collateral Value of Land

3.1 Environment

There are two physical assets in the economy — land and stocks of corn (latter is the numeraire; constituting 'financial' wealth). As in the previous model, corn stocks and land areas are both represented by nonnegative real numbers. Moreover, there are two productive opportunities - agriculture and trade (see further details below). Agents have heterogenous farming ability and asset endowments: agent i has farm ability a_i, initial wealth $w_i \geq 0$ and land endowment $l_{i0} \geq 0$. There are a finite number of agents $i = 1, \ldots, n$. We place no restrictions on the distribution of these endowments.

The timeline is as follows. There are two dates $t = 1, 2$. At start of $t = 1$ markets for land and credit (borrowing/lending corn at $t = 1$, to be repaid at $t = 2$) open; all agents are price-takers. Following land market transactions, agent i ends up with land area l_i, financial wealth $w_i - P(l_i - l_{i0})$ and net loan b_i at the end of $t = 1$. Production (agriculture or trade) inputs are applied. There is no production risk in this environment. At $t = 2$ deterministic production outputs are realized; loans are repaid or defaulted open, which ultimately results in an ex post allocation of corn stocks which are consumed.

Farm production requires effort of owner which is supplied inelastically and at zero cost; no other material inputs or hired labour are needed. Farms are self-cultivated by the owner.[15]

[15]Tenancy in this setting is the same as land ownership: the right to cultivate the land and borrow against the output produced. There is no scope to sell the land at $t = 2$ because that is the last date and land is worthless at or after $t = 2$ — hence exchange rights are irrelevant. Specifically, tenancy would involve leasing of land by the owner to another agent, against payment of rent. If rent is paid upfront (at $t = 1$) it is the same as selling the land to the tenant. An agreement to pay rent at $t = 2$ is credible only if repayment of rent is backed by landlord's right to seize the output produced by the tenant if rent is unpaid: this reduces to a tenancy-cum-credit contract where the tenant borrows the rent from the landlord against the crop

Agent i owning a land plot of size l_i at the end of $t = 1$ produces $a_i l_i^{1-\alpha}$ units of corn at $t = 2$, where $\alpha \in (0, 1)$, so there is diminishing returns to land area.

Agents can pursue farming and trade activities simultaneously. The trade technology is as follows: it does not requite any labor; but uses corn c input or working capital at $t = 1$. This is transported and sold to a distant market at $t = 2$, generating a return of $c+g(c)$ units of corn at $t = 2$, where g is a strictly increasing, concave differentiable function satisfying $g(0) = 0$ and Inada conditions. The latter ensure the agent is better off investing all liquid funds after transacting land into trade, rather than storing them.

Trade working capital can be self-financed (out of trader's own financial wealth) supplemented by borrowing (against land as collateral). As explained in more detail below, financial wealth does not serve as collateral. The credit market functions as follows. An agent can borrow (upto an endogenous credit limit) at $t = 1$, repayable at $t = 2$ at an interest rate which is endogenous. There is ex post moral hazard in loan defaults, which can be circumvented by land as collateral. If a borrower refuses to repay loan at $t = 2$ the lender can seize the harvest on the land cultivated by the borrower (but cannot seize any corn/financial wealth owned by the latter). This generates endogenous borrowing constraints, and forms a source for collateral value for land for agents used to finance trading activities, as explained in the next subsection.

We study competitive equilibrium where all agents take the land price and interest rate as given, land and credit markets clear at $t = 1$. It is not a Walrasian equilibrium because of borrowing and liquidity constraints. It differs from standard models of financial frictions such as Moll 2014, because there are two assets, financial wealth and land, where only land is a form of collateral. Hence the demand for credit indirectly impacts the demand and allocation of land.

3.2 Borrowing Constraints; Individual Agent Optimization Problem

Loan repayments are due immediately prior to the harvest, and repayment takes place in units of corn (just about to be harvested). The underlying enforcement technology is the following. If the borrower defaults, the lender seizes a fraction λ of the output produced by the land at $t = 2$. The remaining fraction $1 - \lambda$ is kept by the borrower. The financial

output as collateral. As landlords have no enforcement advantage over other lenders, this is equivalent to a combination of a land purchase (on the land market) with a credit contract (on the credit market). As combining these two transactions is already possible for an agent, nothing is gained by allowing for such tenancy-cum-credit contracts.

wealth of the borrower cannot be seized. Hence the credit constraint for an agent i is

$$b_i \leq \lambda \frac{a_i l_i^{1-\alpha}}{1+r} \tag{37}$$

where b_i denotes the net amount borrowed and r the interest rate. b_i can be negative, in which case the agent lends rather than borrows, and $-b_i$ is the amount lent. $\lambda \in [0,1]$ is the degree of collateralizabilty of land.[16]

Observe that this formulation of the credit constraint differs from the one made more commonly in the macro literature with financial frictions (e.g., Moll 2014 and references cited there), where financial wealth is the only form of wealth which serves as collateral. Those models do not include land as a separate asset. The rationalization for our formulation is that land (and the output in the pipeline) is an immobile asset which makes it difficult for the borrower to take with him if he absconds, while financial wealth is easier to move. This is necessary for land to serve as a form of collateral, and accordingly for the demand for land to include a collateral motive which finances an alternative (trade) activity. Our formulation is similar to models like Kiyotaki and Moore 1997 where land is a form of collateral.

Letting P denote the land price, the amount available for trade capital to agent i equals

$$c_i = w_i + Pl_{i0} + b_i - Pl_i \tag{38}$$

the sum of financial wealth, value of land endowment and borrowing, less the value of land purchased at $t = 1$. Since trading scale c_i has to be non-negative, (38) reduces to the liquidity constraint

$$W_i + b_i - Pl_i \geq 0 \tag{39}$$

where $W_i \equiv w_i + Pl_{i0}$, the value of the agent's initial endowment.

Date 2 consumption equals the sum of returns from agriculture and trade, less loan repayments:

$$\begin{aligned} & a_i l_i^{1-\alpha} + c_i + g(c_i) - (1+r)b_i \\ = {} & [a_i l_i^{1-\alpha} - Pl_i] + [g(W_i + b_i - Pl_i) - rb_i] + W_i \end{aligned} \tag{40}$$

the sum of net returns from agriculture and trade after accounting for cost of inputs, and initial endowment valued at the equilibrium land price.

[16]Implicit in this formulation is the assumption that the ability of the farmer is publicly known. A weaker assumption is that lenders have some information about the borrower's ability, and $\lambda a_i l_i^{1-\alpha}$ is the output they expect to be able to appropriate in the event of default. See the last subsection of this model for further discussion of this extension.

Therefore agent i faces the following optimization problem. Choose $l_i \geq 0$ and b_i to maximize (41) subject to borrowing and liquidity constraints (37, 39), taking P and r as given. Observe that b could be negative, in which case the agent becomes a net lender.

3.3 Competitive Equilibrium (CE)

A **competitive equilibrium (CE)** is an allocation $(\{l_i, b_i\}_{i=1,2,..})$, a land price P and an interest rate r such that:

(a) agent i selects (l_i, b_i) to maximize $[a_i l_i^{1-\alpha} - Pl_i] + [g(W_i + b_i - Pl_i) - rb_i]$ subject to $W_i + b_i - Pl_i \geq 0$ and $b_i \leq \lambda \frac{a_i l_i^{1-\alpha}}{1+r}$, where W_i denotes $w_i + Pl_{i0}$.

(b) land and credit markets clear: $\sum_i l_i = \sum_i l_{i0}$ and $\sum_i b_i = 0$.

A CE **features no land misallocation** if $(l_1, l_2, ..)$ maximizes $\sum_i a_i l_i^{1-\alpha}$ subject to $\sum_i l_i = \sum_i l_{i0}$, i.e., marginal product of land $a_i l_i^{1-\alpha}$ is equalized across all i.

3.4 Equilibrium Misallocation

Since all agents are price takers and take P, r as given, studying the solution to the agent optimization problem (a) for any given P, r provides insight into the nature of misallocation that arises. We break down this optimization problem into a two step problem. First take a choice of land area l_i, and optimize over the choice of credit b_i to maximize the net returns from trade $g(W_i + b_i - Pl_i) - rb_i]$ given l_i and the borrowing constraint. The solution to this generates the collateral value of land, the contribution of land owned to trade profits. Using this, at the second stage optimize over l_i taking into account the sum of profits from agriculture $a_i l_i^{1-\alpha} - Pl_i$ and the collateral value of land.

To simplify notation, we drop the agent index i and consider a representative agent with ability a, endowments w, l_0, who chooses (l, b) to maximize $[al^{1-\alpha} - Pl] + [g(W + b - Pl) - rb]$ subject to $W + b - Pl \geq 0$ and $b \leq \lambda \frac{al^{1-\alpha}}{1+r}$, where W denotes $w + Pl_0$. For most part we will not carry the notation for P, r, but it should be kept in mind that W does depend on the endogenous land price P. Of course, the agent being a price taker takes the value of W as given.

3.4.1 Step 1: Optimal Trade Credit, given Land Area

To set up the first step, we first need to restrict attention to a land area l that is feasible for the agent, given the borrowing and liquidity constraints. These two constraints imply

$Pl \leq W + b \leq W + \frac{al^{1-\alpha}}{1+r}$, so a feasible land area choice is characterized by

$$Pl \leq W + \frac{al^{1-\alpha}}{1+r} \tag{41}$$

Let $\hat{l}(a, W)$ denote the solution for l to the equality version of (41). Clearly this is strictly positive for all $W \geq 0$ and increasing in W. Then the land area feasibility constraint is written as

$$l \leq \hat{l}(a, W) \tag{42}$$

Given any l satisfying (42), the first step problem is to select b to maximize $g(W + b - Pl) - rb$ subject to $Pl - W \leq b \leq \lambda \frac{al^{1-\alpha}}{1+r}$.

It is convenient to reformulate this by redefining the choice variable from b to $s \equiv b - Pl$, so the problem is now: Choose s to maximize

$$[g(W + s) - rs] - rPl \tag{43}$$

subject to

$$-W \leq s \leq \lambda \frac{al^{1-\alpha}}{1+r} - Pl. \tag{44}$$

s represents the net addition to trade capital besides own wealth W, taking into account both borrowing b and the amount spent on purchasing land l. The collateral role of land helps bolster what the agent can borrow, but it also reduces the agent's own wealth by the need to pay for the land acquired (or not sold). The difference between these two defines the extent to which land ownership allows the agent to supplement trade capital, represented by the upper bound $\lambda \frac{al^{1-\alpha}}{1+r} - Pl$ to s. (Observe also that the lower bound constraint on s in (44) will never bind, owing to the Inada property of g. Hence it can be dropped from now onwards.)

The other advantage of this reformulation is that as l is given at the first step, the above problem reduces to maximization of $[g(W + s) - rs]$ subject to (44); we then subtract rPl from the solution. The former objective function no longer depends on land area l, only on the agent's wealth. In particular, land area matters only if the borrowing constraint is binding, and its effect operates by determining the borrowing limit. The following Lemma provides a detailed characterization of the solution to the first step problem, which helps understand the collateral value of land. For this we need the following notation.

Let $Q(l; a, W)$ denote the maximum value of trade profit $[g(W+s) - rs]$ over choices of s satisfying (44). Use S^* to denote the unconstrained optimal level of trade finance, satisfying $g'(S^*) = r$.

Next, use $\bar{F}(a)$ to denote the maximized achievable agriculture profit for an agent with ability $F(l,a) = al^{1-\alpha} - (1+r)Pl$ with respect to choice of l. This maximum is achieved at $l_F(a) \equiv [\frac{(1-\alpha)a}{(1+r)P}]^{\frac{1}{\alpha}}$. Clearly, $F(l,a)$ is strictly concave in l, rising from $l=0$ to $l=l_F(a)$, and falling thereafter.

Lemma 4 *(a) If $W \geq S^*$, the agent chooses trade finance level S^*, lends $W - S^*$ thereby achieving trade profit*

$$Q(l;a,W) = g(S^*) - rS^* + rW \tag{45}$$

which is independent of l.

(b) If $W < S^* - \frac{\bar{F}(\lambda a)}{1+r}$, trade finance level S^* is unachievable, and the borrowing constraint binds for every feasible choice of l. Given l, the constrained optimal trade finance equals $W + \hat{s}(l,\lambda a)$ where $\hat{s}(l,\lambda a) \equiv \frac{F(l,\lambda a)}{1+r}$, resulting in trade profit

$$Q(l,a,W) = g(W + \frac{F(l,\lambda a)}{1+r}) - r\frac{F(l,\lambda a)}{1+r}. \tag{46}$$

and the marginal collateral value of land is positive over the range $l < l_F(\lambda a)$ and negative thereafter.

(c) If $S^* - \frac{\bar{F}(\lambda a)}{1+r} \leq W < S^*$, the unconstrained trade finance level S^* is achievable for an interval of land areas $[l_1(a,W), l_2(a,W)]$ containing $l_F(\lambda a)$; over this range Q is given by (45). It is not achievable outside this range, and Q is given by (46). The marginal collateral value of land is positive for $l < l_1(a,W)$, zero over $[l_1(a,W), l_2(a,W)]$ and negative thereafter.

The proof is straightforward and thus omitted. The solution is intuitive. Part (a) considers agents with initial wealth sufficient to finance S^* the unconstrained optimal level of trade finance. These agents lend rather than borrow, and have no need to hold land to augment credit access. So for this category of wealthy agents land has no collateral value.

Part (b) describes an upper bound on wealth below which agents can never achieve the unconstrained trade finance level S^*, no matter how much land they own. For these wealth constrained agents, the borrowing constraint always binds, and how much land they own matters for the trade profits they earn. *The key point to note is that the marginal collateral value of land for these agents is positive for small land areas owned upto $l_F(\lambda a)$, and negative thereafter.* This is because increasing land area both allows these agents to borrow more, but also reduces their own wealth after accounting for the cost of the land. The former effect dominates over an initial range, but is dominated by the land cost effect thereafter.

Finally, part (c) describes the intermediate category of wealth for whom the borrowing constraint does not bind over an intermediate range of land areas, but does bind outside it. For them the marginal collateral value of land is initially positive, zero over the intermediate range and negative thereafter.

3.4.2 Step 2: Optimal Land Area

The optimal trade profit achieved at the first step equals $Q(l; a, W) - rPl$. The first step problem can therefore be posed as follows. Choose l to maximize the sum of profit from agriculture and trade

$$F(l, a) + Q(l; a, W) \tag{47}$$

subject to (42).

3.5 Equilibrium Misallocation

The nature of optimal land demands and how they vary across agents of disparate types at any given set of prices will obviously apply to any equilibrium set of prices, where land areas allocated will equal areas demanded by respective agents. So demand heterogeneity patterns serve to determine the nature of land misallocation resulting in any equilibrium.

Proposition 5 *If land is perfectly collateralizable ($\lambda = 1$) every agent with ability a will select $l = l_F(a)$. Hence there will be no misallocation in any competitive equilibrium.*

Proof: For agents with $W \geq S^*$ this is obvious. For all other agents, observe that Q and F are both maximized at $l = l_F(a)$, and for F this is the unique maximizer. ■

With $\lambda = 1$, the borrowing limit is proportional to agricultural profit, so the collateral value of land is maximized at the same level of $l = l_F(a)$ that maximizes agricultural profit, and there is no tradeoff between profit from agriculture and trade. Interestingly, this applies to all agents, irrespective of whether or not they are wealth constrained.

Next consider the case when $\lambda < 1$, where there is misallocation, described as follows.

Proposition 6 *Suppose land is imperfectly collateralizable ($\lambda < 1$). Let $l(a, W)$ denote the optimal land area.*

(a) If $W \geq S^*$, $l(a, W) = l_F(a)$.

(b) If $W < S^* - \frac{\bar{F}(\lambda a)}{1+r}$, $l(a, W)$ lies in the open interval $(l_F(\lambda a), l_F(a))$, and is increasing in both a and W.

(c) If $S^* - \frac{\bar{F}(\lambda a)}{1+r} \leq W < S^*$, $l(a,W)$ lies in the half open interval $(l_F(\lambda a), l_F(a)]$, is increasing in a and weakly increasing in W.

Consequently there will be misallocation in any equilibrium in which there are some agents in groups (a) and (b) respectively.

Proof: (a) If $W > S^*$ trade profit Q is independent of l, so it is optimal for the agent to select $l_F(a)$ which maximizes $F(l,a)$, since it satisfies (41) and hence also (42).

(b) In this case, both agricultural and trade profit are rising in l until $l = l_F(\lambda a)$ which is strictly smaller than $\hat{l}(a,W)$. Thus $l_F(\lambda a)$ and slightly higher levels of l are feasible. Raising l slightly above $l_F(\lambda a)$ results in a first order increase in agricultural profit, while there is a second-order decline in trade profit, implying that total profit increases. A converse argument implies the optimal l must be strictly lower than $l_F(a)$ if the latter is feasible. And the same conclusion holds it is infeasible: $l(a,W) \leq \hat{l}(a,W) < l_F(a)$. The first order condition $F_l(l,a) + Q_l(l,a,W) = 0$ characterizes any interior $l(a,W) < \hat{l}(a,W)$, where Q is given by (46). Since $F_l = (1-\alpha)al^{-\alpha}$ is increasing in a and independent, it suffices to show that $Q_{la} > 0$ and $Q_{lW} > 0$ at $l = l(a,W)$. From (46) we see that

$$Q_l(l;a,W) = [g'(W + \frac{F(l,\lambda a)}{1+r}) - r]\frac{F_l(l,\lambda a)}{1+r} \tag{48}$$

Therefore

$$\begin{aligned} Q_{la}(l;a,W) &= g''(W + \frac{F(l,\lambda a)}{1+r})\frac{F_a(l,\lambda a)}{1+r}\frac{F_l(l,\lambda a)}{1+r} \\ &\quad + [g'(W + \frac{F(l,\lambda a)}{1+r}) - r]\frac{F_{la}(l,\lambda a)}{1+r} \end{aligned} \tag{49}$$

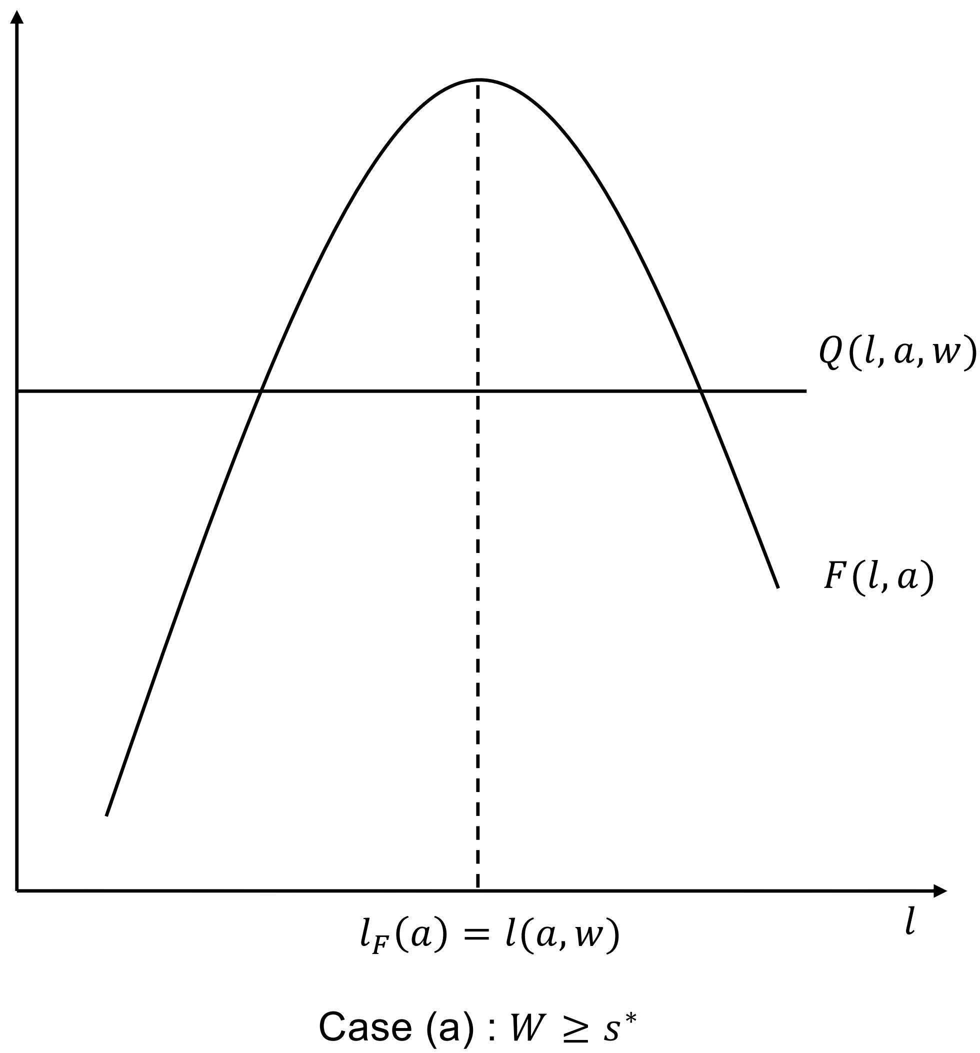
which is positive at any $l > l_F(\lambda a)$ because this implies $F_l(l,\lambda a) < 0$, while $g'' < 0$, $F_a > 0$, $F_{la}(l,\lambda a) > 0$ holds everywhere. Moreover,

$$Q_{lW} = g''(W + \frac{F(l,\lambda a)}{1+r})F_l(l,\lambda a) \tag{50}$$

is positive at any $l > l_F(\lambda a)$. Hence $l(a,W)$ is increasing in both arguments at any interior optimum.

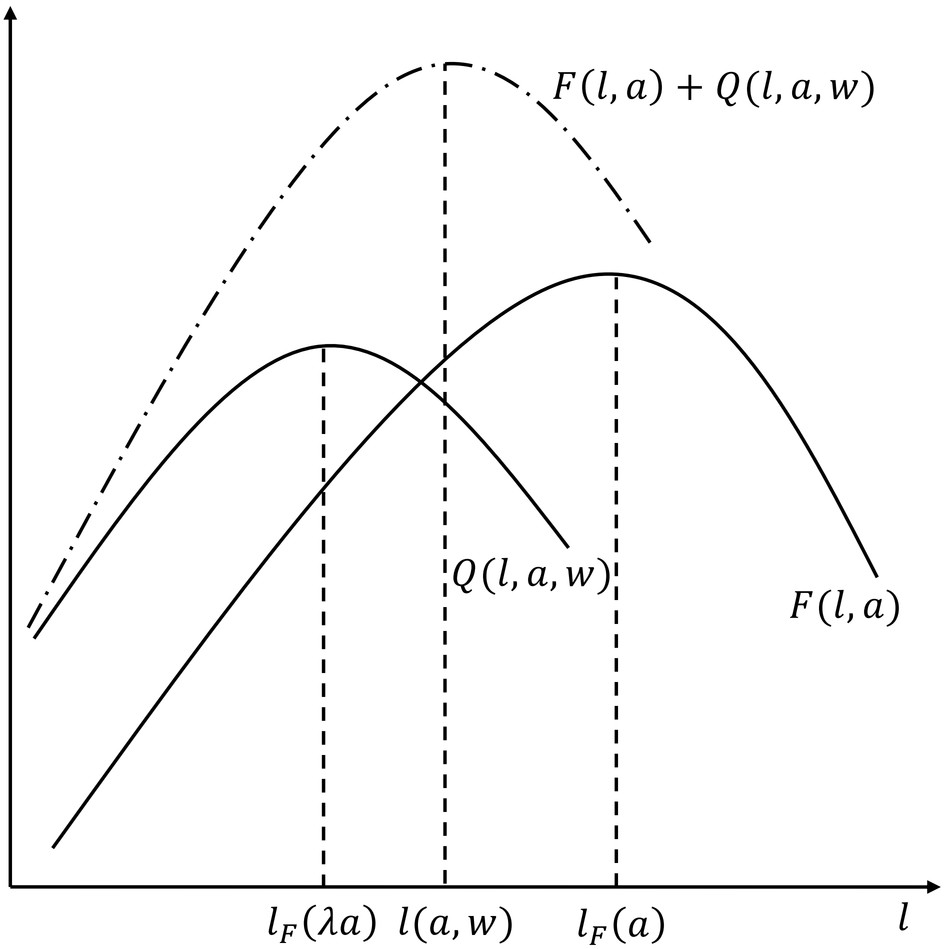
For (c), the only modification of arguments in (b) above arise because Q could be flat rather than strictly concave in l over a range of intermediate values of l which contains $l_F(\lambda a)$. It must continue to be optimal to raise l slightly above $l_F(\lambda a)$. However, if the range over which Q is flat includes $l_F(a)$, it would be optimal to select $l = l_F(a)$. Q_{la} and Q_{lW} will be zero if the optimal l lies in the range where Q is flat, while $F_{la} > 0$ and F_l is independent of W. Hence $l(a, W)$ will be sttrictly increasing in a and weakly increasing in W, starting from any interior optimum. ■

FIGURE 3: **Case (a): Wealth Unconstrained Agents**



Wealthy agents in group (a) select first-best land area as this maximizes agricultural profits, while the marginal collateral value of land is zero as these agents are net lenders rather than borrowers (See Figure 3). Poor agents in group (b) (illustrated in Figure 4) are net borrowers and credit constrained: they value land for its collateral value. However as shown in Lemma 1, trade profits are maximized as $l = l_F(\lambda a)$, which is smaller than the first-best land area that maximizes agricultural profits, when land is imperfectly collateralizable. Hence they select a land size that is intermediate between $l_F(\lambda a)$ and $l_F(a)$, where Q_l the marginal collateral value of land is negative, while F_l the gradient of agricultural profit is

FIGURE 4: **Case (b): Wealth Constrained Agents**



Case (b) : $W < s^* - \frac{\overline{F}(\lambda a)}{1+r}$, $\lambda < 1$

positive. They end up with a land area smaller than first-best, by an extent that depends on their wealth. The lower their wealth, the closer is their land area to $l_F(\lambda a)$ and further away from the first-best level. Agents in group (c) are qualitatively similar to those in (b), except that the wealthiest among them may actually attain the first best land area.

The condition for misallocation to occur in an equilibrium is that there is enough wealth heterogeneity that there are some agents in groups (a) and (b) respectively. As wealth depends on the land price and the wealth threshold at which borrowing constraints bind depends on the interest rate, both of which are endogenously determined in equilibrium, this condition cannot be verified directly from the economy's fundamentals. However, with sufficient heterogeneity of land endowments, the credit market equilibrium will endogenously sort agents into lenders in group (a) and borrrowers in groups (b) and (c). So in an economy where most of the land and financial wealth is concentrated among a few agents, and others who own none of either type of asset, the former will sort into group (a) and the latter into group (b).

The nature of the misallocation therefore ends up qualitatively similar to that in standard models of financial frictions such as Buera et al 2011 or Moll 2014. Agents with high ability and low wealth end up with less land than those with low ability and high wealth. The extent of misallocation depends on how ability and wealth are correlated: it tends to be small if this correlation is positive and large. And as shown in Proposition 5 it also depends on λ: there is no misallocation if land is perfectly collateralizable, irrespective of how unequally assets are distributed. An efficient and active credit market with high enough λ can achieve the first-best allocation, despite the existence of substantial wealth heterogeneity, co-existence of poor agents with binding credit constraints that borrow from a few wealthy lenders, possibly even at high interest rates.

3.6 Extension to Asymmetric Information on the Credit Market

Part of what drives the preceding result is the assumption that lenders are perfectly informed about borrowers' farming abilities, besides the capacity of lenders to seize a large enough fraction of the output of borrowers that default. The absence of asymmetric information of borrower ability allows a high ability but extremely poor agent to be able to borrow enough that enables it to attain the first-best. Lenders are therefore able to calibrate borrowing limits accurately to borrower ability.[17] An interesting extension of the model is to the case where lenders have no capacity to discriminate between borrowers of varying abilities, and every agent faces a uniform borrowing limit that does not vary with ability:

$$b_i \leq \bar{a}\frac{l_i^{1-\alpha}}{1+r} \tag{51}$$

instead of (37), where $\bar{a}$ denotes the average ability in the population. This corresponds to a context where lenders can seize all the output of a defaulting borrower (so $\lambda = 1$), but have no information about the borrower's productivity so end up 'pooling'.

What does our model predict in this case? Credit access and therefore optimal trade profit $Q(l; .)$ no longer depends on the agent's ability. The collateral value of land will be the

[17]In the absence of perfectly accurate information, lenders' assessments could still vary coarsely with borrower ability: this is partly what may cause λ to be smaller than one. However our formulation of the borrowing constraint would then require expected ability to equal λa and thus still remain proportional to true ability.

same across agents of disparate abilities, and will correspond to the collateral value for an agent with average ability $\bar{a}$. Trade profits for wealth constrained borrowers will therefore be maximized at $l_F(\bar{a})$, irrespective of their ability. Hence the collateral value of land would be independent of ability, in contrast to the security value of land. Relative to the case where ability is known by lenders as in Proposition 5, the land area demanded will therefore increase for below-average ability agents, remain the same for an average ability agent, and fall for high ability agents. In other words, there will be misallocation, with low ability agents selecting land areas larger than the first-best (corresponding to their ability level), with the converse true for high ability agents. Since F_{la} is still positive while Q_{la} is now zero, higher ability agents will still end up with higher land areas. But the land allocation will become skewed in favor of low ability agents.

On the other hand, the pattern of wealth effects will be similar to the security value of land. (50) will be replaced by

$$Q_{lW} = g^{''}(W + \frac{F(l,\bar{a})}{1+r})F_l(l,\bar{a}) \tag{52}$$

and $F_l(l,\bar{a})$ will be positive for low ability agents, zero for the average ability agent, and negative for high ability agents. So wealth effects will be negative (positive) for below (resp. above) average ability agents. In this respect, this version of the collateral value model generates the same prediction as the security value model.

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